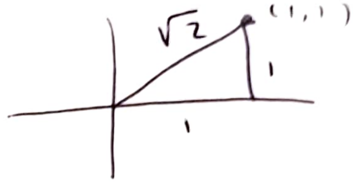
1. a) $\bar{z}_1 z_2 = (-1-i)\sqrt{2}\, e^{\pi/4\, i}$

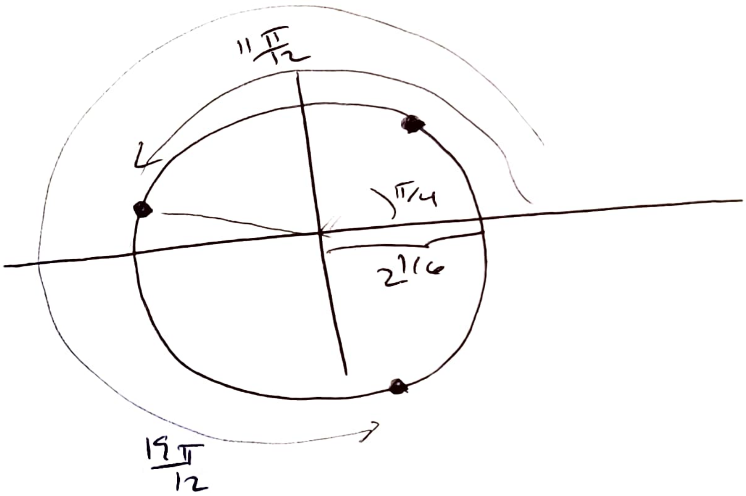


$= (-1-i)(1+i)$

$= -1 - i - i + 1 = -2i$

b) $z_1^{1/3} = \left(\sqrt{2}\, e^{(\frac{3\pi}{4} + 2n\pi) i}\right)^{1/3} = 2^{1/6}\, e^{\left(\frac{\pi}{4} + \frac{2n\pi}{3}\right) i}$

$= \left\{ 2^{1/6} e^{\frac{\pi}{4} i},\ 2^{1/6} e^{\frac{11}{12}\pi i},\ 2^{1/6} e^{\frac{19}{12}\pi i} \right\}$



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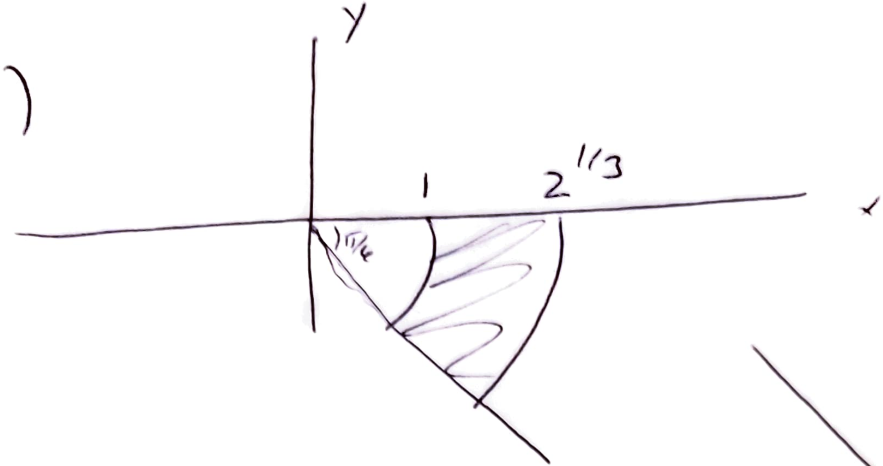
2 a) $\lim\limits_{z \to i} \dfrac{(\overline{z-i})^2}{(z-i)^2} = \lim\limits_{z_1 \to 0} \dfrac{\bar{z}_1^{\,2}}{z_1^{\,2}} = \lim\limits_{z_2 \to 0} \dfrac{\bar{z}_2}{z_2}$

DNE To prove this consider limits on vertical & horizontal lines. Show they do not agree.

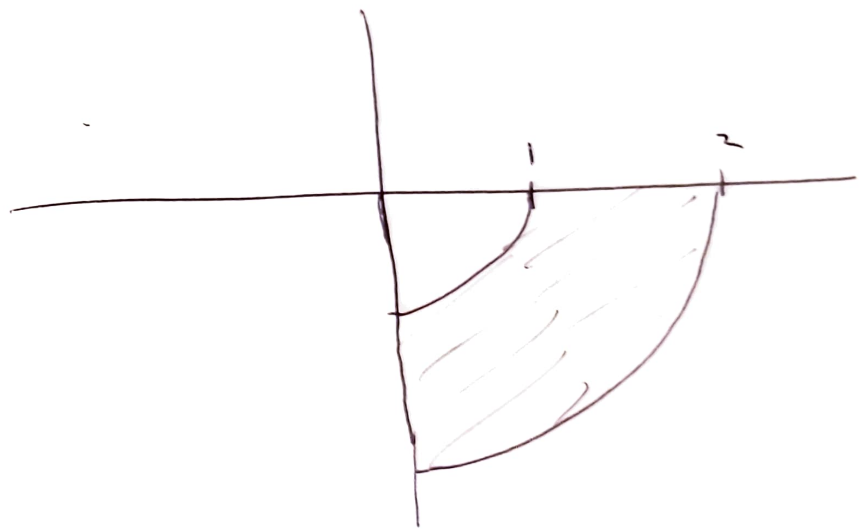
b) $\lim\limits_{z \to e^{\frac{\pi}{4} i}} \left[\dfrac{z^2 - i}{z^4 + 1} = \dfrac{z^2 - i}{(z^2+i)(z^2-i)} \right] = \lim\limits_{z \to e^{\frac{\pi}{4} i}} \dfrac{1}{z^2 + i}$

$= \dfrac{1}{e^{\pi/2\, i} + i} = \boxed{\dfrac{1}{2i}}$.

3. a)



b) $w = z^3 = r^3 e^{3i\theta}$



4. $\boxed{g(z) = \text{const}}$

Proof: $g(z) = u + iv$. $u = \text{const}$.

$\Rightarrow 0 = u_x = v_y$ $\Rightarrow$ $v = \text{const}$.

$0 = u_y = -v_x$

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5. $f = x^2 + y^2 + (x-y)i$

$u_x = 2x$, $u_y = 2y$, $v_x = 1$, $v_y = -1$

$u_x = v_y \Rightarrow x = -\frac{1}{2}$

$u_y = -v_x \Rightarrow y = -\frac{1}{2}$

$f'(-\frac{1}{2} - \frac{1}{2}i) = u_x + iv_x = -1 + i$

So f is differentiable only at $(-\frac{1}{2}, -\frac{1}{2})$.

Not holomorphic there since need a neighborhood for this to be true.

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6. $\log(1+i)^2 = \log(\sqrt{2}e^{i\pi/4})^2 = \log 2e^{i\pi/2} = \boxed{\ln 2 + i(-\frac{3\pi}{2})}$

$2\log(1+i) = 2\log\sqrt{2}e^{i\pi/4} = 2(\ln\sqrt{2} + i\pi/4)$

$= \ln 2 + i\pi/2$ Not equal.